Clock Dynamics Modeling in High Precision Geolocation

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I. Generalities

Characterization of the clock dynamics in a cellular phone is extremely critical to high precision geolocation. The random fluctuations or deviations in oscillators can often be characterized by power law spectra. The typical names given to the various noise types that afflict a generic oscillator are:

- white-noise Phase Modulation,
- flicker-noise Phase Modulation,
- white-noise Frequency Modulation,
- flicker-noise Frequency Modulation,
- random-walk Frequency Modulation.

In essence this means that the Power Spectral Density of the `frequency deviation` for an oscillator is well approximated by

$$S_y(f) = \sum_{n=-2}^{2} h_n f^n,$$

where h_n are the Allan parameters. The classical way of relating these noise sources to a statistical model was explaned by David W. Allan [1] and generally accepted by the GPS community [2].

II. Statistical Modeling of Clock Errors

The classical `random FM plus random-walk FM` model states that bias evolution at time t_k defined $\tau(t_k)$ and drift at time t_k defined $f(t_k)$ of a clock can be modeled as

$$\boxed{\text{bias evolution:} \quad \tau(t_k) = \tau(t_{k-1}) + w_p(k) + \Delta \cdot f(t_{k-1})} \tag{1}$$

$$\boxed{\text{drift evolution:} \quad f(t_k) = f(t_{k-1}) + w_f(k)} \tag{2}$$

where $w_f(k)$ and $w_p(k)$ are zero-mean Gaussian processes with statistics related to the Allan parameters and $\Delta = t_k - t_{k-1}$. The variances of $w_f(k)$ $w_p(k)$ are

$$E\{w_f(k)w_f(k)\} = A \cdot \Delta + B \cdot \Delta^3/3,$$

$$E\{w_p(k)w_f(k)\} = B \cdot \Delta^2/2,$$

$$E\{w_p(k)w_p(k)\} = B \cdot \Delta,$$

where

$$A = 2 \cdot h_0,$$

$$B = 8 \cdot \pi^2 \cdot h_{-2}.$$

The two parameters h_0 and h_{-2} are the random FM and random-walk FM parameters explained and introduced by Allan in his remarkable work [1]. The relationship between these parameters and the Allan variance of a clock is:

$$\boxed{\sigma_y^2(\tau) \simeq \frac{h_0}{2 \cdot \tau} + 2/3\pi^2 \cdot h_{-2} \cdot \tau.} \quad (3)$$

III. Parameters of the Model

A low quality oscillator of the type used in cell phones might have:

`Low performance Oscillator (Crystal)`

$$h_0 = 2 \times 10^{-19} \ \ sec^2/sec$$

$$h_{-2} = 2 \times 10^{-20} \ \ sec^2/sec^3$$

or

$$h_0 = (2.2 \pm 0.4) \times 10^{-19} \ \ sec^2/sec$$

$$h_{-2} = 2 \times 10^{-26} \ \ sec^2/sec^3$$

A high performance oscillator of the type used in Cellular Base-Stations might be characterized by the following parameters:

`High performance Oscillator (Atomic)`

$$h_0 = 2 \times 10^{-20} \ \ sec^2/sec$$

$$h_{-2} = 2 \times 10^{-29} \ \ sec^2/sec^3$$

References

[1] Allan D. W., *"Time and Frequency (Time-Domain) Characterization Estimation, and Prediction of Precision Clocks and Oscillators"* in *IEEE Trans. on Ultrasonics, Ferroelectrics, and Frequency Control* November 1987.

[2] Spilker J. J. Jr., *"GPS Navigation Data"* in *"Global Positioning System: Theory and Applications, Vol. I"* published by AIAA, Inc., 1980.

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