Martone Radio Technology, Inc.

Artemis is "Close to Theoretical"

The performance of a geolocation system and the impact of geometry as well as the impact of the measurements noise are well characterized by the Cramer-Rao lower bound, the theoretical limit in an unobstructed envirornment. Artemis performance is close to the theoretical limit in urban environments.

Assume an East-North-Up referenced coordinate system and consider a Target at position (X, Y). The bearing measurement system provides measurements Angle of Arrival (AOA) measurements $\theta(t)$, the platform is moving at coordinates $X_p(t)$, $Y_p(t)$. The relationship between measurements and coordinates is:

$$\hat{\theta}(t) = \text{atan}\frac{X - X_p(t)}{Y - Y_p(t)} + \texttt{error}.$$

This system of equations can be linearized and solved. The characterization of the error terms is something that escapes human understanding. Since the human brain has a tendency to fit physical phenomena to what really it can quantify and express we allow ourselves to characterize the error term made of a Gaussian term and bias:

$$\texttt{error} = \phi + n(t).$$

This is far from reality and it can NOT possibly be demonstrated that the random process $n(t)$ is stationary. To simplify the discussion for the purpose of this technical note we will proceed with the "academic" model of disturbance. The target position must be **observable**: this means that the solution is unique, in other words the measurements generated by the platform path should not generate two or more target positions. If we define (X_a, Y_a, ϕ_a) and (X_b, Y_b, ϕ_b) two target positions and biases, we see after straightforward algebra that

$$\begin{aligned}[X_p(t) - K_1]^2 + [Y_p(t) - K_2]^2 \\ = (X_a - K_1)^2 + (Y_a - K_2)^2 \end{aligned} \quad (1)$$

where

$$K_1 = [X_a + X_b + K(Y_b - Y_a)]/2,$$

$$K_2 = [Y_a + Y_b + K(X_b - X_a)]/2.$$

and $K = \texttt{ctg}(\phi_a + \phi_b)$. This means that the target is observable if and only if the platform path does not satisfy (1) for any constants K_1 and K_2. (1) is an arbitrary circle passing through the target and in the limiting case where $K \rightarrow \infty$ it degenerates to a straight line passing also through the target. Evidently the condition for observability is that the path should not be on a circle through the target and not along the line of sight to the target.

The Cramer-Rao Lower Bound (CRLB) in the case of a biased measurement set obtained at t_i $i = 1, 2, ..., N$ is

$$\mathbf{S} = \sigma^2 \left(\mathbf{V} + \frac{\mathbf{V}\ \mathbf{f}\ \mathbf{f}^T\ \mathbf{V}}{\sigma/\sigma_\phi + N - \mathbf{f}^T\mathbf{V}\mathbf{f}} \right), \quad (2)$$

where σ^2 is the variance of the measurement noise assumed uncorrelated, σ_ϕ^2 is the variance of the bias ϕ assumed zero-mean and Gaussian and we have used the notation

$$\mathbf{f} = \mathbf{G}^T\mathbf{1}_N, \qquad \mathbf{V} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}, \quad (3)$$

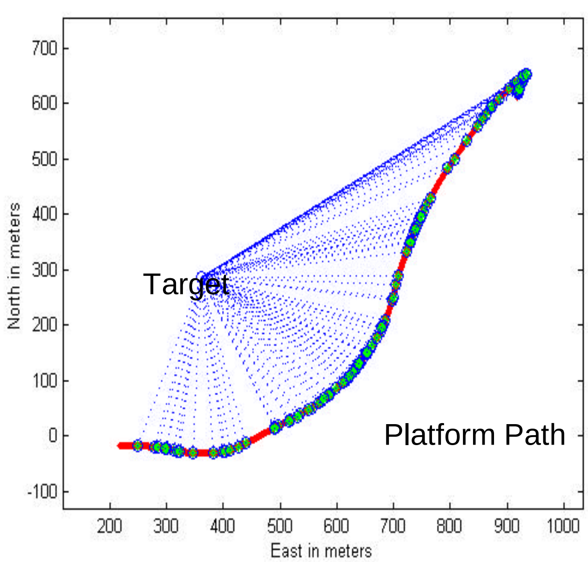
with

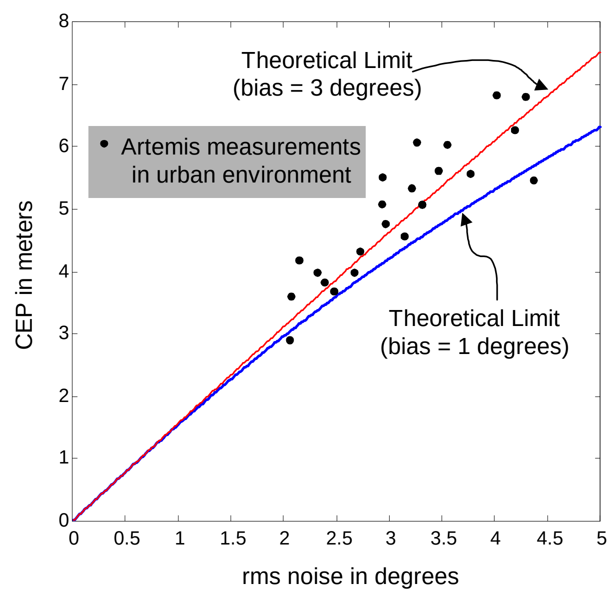
$$\mathbf{G} = \begin{bmatrix} -\Delta_{Y1}/r_1^2 & \dots & -\Delta_{YN}/r_N^2 \\ -\Delta_{X1}/r_1^2 & \dots & -\Delta_{XN}/r_N^2 \end{bmatrix}^T \quad (4)$$

$r_i^2 = \Delta_X i^2 + \Delta_Y i^2$ and $\Delta_{Xi} = X - X_p(t_i)$, $\Delta_{Yi} = Y - Y_p(t_i)$.

The CEP is a crude measure of accuracy as opposed to the more elaboarate ellipse of uncertainty. The CEP is however valid only in the unbiased case and can be aaproximated as

$$CEP = 0.75\sqrt{\texttt{Trace}[\mathbf{S}]}. \quad (5)$$





2410 Camino Ramon , Suite 180, San Ramon, CA 94583
Ph: 925 806 0413 Fax: 925 806 0382 www.martoneradiotech.com